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Internal Wave Maker for Navier-Stokes Equations in a Three-Dimensional Numerical Model

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ABSTRACT

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Recently, numerical simulations of water surface waves using the Navier-Stokes equations models have been increased since those models are well known as one of the numerical models capable of overcoming the limitations of nonlinearity, dispersion and wave breaking. For the Navier-Stokes equations model, an internal wave maker using the mass source function has been employed to generate various types of waves. However, the method has been applied to vertically two-dimensional numerical models and has not been expanded to three dimensions yet. In this study, the internal wave maker for the Navier-Stokes equations is employed to generate target waves in the three-dimensional numerical wave tank, NEWTANK. The model is first verified by applying to simple numerical tests in two dimensions. The model is then applied to predict wave transformation on a variable bathymetry in two dimensions and expanded to three dimensions. Numerical results are compared with available laboratory measurements and flow characteristics are analyzed in detail.

ADDITIONAL INDEX WORDS: *Solitary wave, Runup and rundown, Conical island*

INTRODUCTION

Many numerical models have been used to study wave behaviors in coastal areas. The transformation of multi-directional waves can be numerically simulated by solving a boundary value problem. To solve the boundary value problem, the wave energy is required to be generated precisely at the offshore boundary. Then, governing equations, which are generally partial differential equations such as the shallow water equations, the Boussinesq equations and also the Navier-Stokes equations, should be solved in both time and space by using appropriate numerical techniques. However, different types of problems and some limitation are induced when numerical models are employed to predict wave transformation. One of the key problems occurring in simulating wave motion is re-reflection to the computation domain at the incident boundary. Many numerical techniques have been developed to deal with this problem and the internal wave generating-absorbing boundary condition has been commonly used in numerical wave models.

In depth-averaged equations wave models, the method of characteristics is generally employed to separate incident and reflected waves. For example, based on one-dimensional shallow water equations, Kobayashi *et al.* (1987) developed an approximate boundary condition for absorbing the reflected waves while sending in the incident waves at the same time. Following the approach, a similar type of wave absorbing-generating boundary condition is developed for a two-dimensional shallow water equations model. Since these methods are based on shallow water assumptions, the accuracy problem has been occurred when the methods are applied to dealing with short period waves. All of these approaches assume the linear superposition of incident and reflected waves at the wave generating-absorbing boundary. Thus, these can only be employed to small amplitude waves. Furthermore, Wei and Kirby (1995) found that this type of boundary condition can lead to the accumulation of errors that may eventually contaminate numerical results in the entire domain when a long time simulation is performed (Lin and Liu, 1999).

Recently, numerical simulations of water surface waves using the Navier-Stokes equation models have been common since these models are much more superior than the depth-averaged wave models. The Navier-Stokes equation-based models employ the VOF (volume of fluid) method, therefore, those can manage interfaces between multiple phases, i.e., water and air. In addition to that, the models can solve the complete momentum equation, resolving vertical components as well as nonlinear terms and viscous and turbulent stresses. Therefore, those models are known as one of the numerical models capable of overcoming the limitations of nonlinearity, dispersion and wave breaking, which most of the conventional wave models cannot do. Although application of the model is still in an early stage, studies on water surface waves by using the Navier-Stokes equations models employing the VOF scheme have been increasing (Choi and Yoon, 2009).

It is even more difficult to treat the wave absorbing-generating boundary for a numerical model based on Navier-Stokes equations or Reynolds averaged Navier-Stokes (RANS) equations, since the model should describe the vertical velocity distribution as well as the free surface displacement at the boundary (Miyata 1986; Lin and Liu 1998). Therefore, the model is much more sensitive to the errors from the boundary than the depth-averaged equations models, such as the shallow water equations model and the

Boussinesq equations model (Lin and Liu, 1999). For the Navier-Stokes or RANS equation model with the VOF method, Lin and Liu (1999) proposed the internal wave maker using mass source function of the continuity equation and this wave maker has been used to generate various types of waves. The method has been applied in several numerical tests and succeeded in predicting wave transformation in the numerical wave tank (Garcia *et al.*, 2004; Lara *et al.*, 2006; Lin and Karunarathna, 2007). However, all tests are performed in two dimensions and have not been expanded to three dimensions yet.

In this study, the internal wave maker for the Navier-Stokes equations which is using mass source function of the continuity equation is employed to generate targeted linear and nonlinear waves in the three-dimensional numerical model. By applying the present method to the three-dimensional numerical wave tank, NEWTANK, different numerical tests are performed in two and three dimensions.

NUMERICAL MODEL

Mathematical model

The motions of an incompressible flow are governed by the Navier-Stokes equations and the continuity equation.

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p_i}{\partial x_i} + g_i + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} \qquad (2)$$

where $i, j = 1,2,3$ for the three-dimensional flows, ρ denotes the density, p is the pressure, u_i and g_i represent $i-$th component of the velocity vector and the gravitational acceleration, respectively.

The direct numerical simulation (DNS) to NSE for turbulent flows at high Reynolds number is computationally too expensive. As an alternative, the large eddy simulation (LES) approach (Deardorff, 1970), which solves the large scale eddy motions according to the spatially averaged Navier-Stokes (SANS) equations and models the small-scale turbulent fluctuations, becomes attractive. In the LES approach, the top-hat space filter (Pope, 2000) is applied to the NSE and the resulting filtered equations of motions are as:

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0 \qquad (3)$$

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \overline{u_i u_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p_i}}{\partial x_i} + g_i + \frac{1}{\rho}\frac{\partial \overline{\tau_{ij}^r}}{\partial x_j} \qquad (4)$$

where $\overline{p}$ denotes the filtered pressure, $\overline{u_i}$ and $\overline{\tau_{ij}^r}$ represent filtered velocities and the viscous stress of the filtered velocity field, respectively. The viscous stress terms are modelled by the Smagorinsky SGS model (Smagorinsky, 1963).

Numerical solver for NSE

In this model, the governing equations are solved by the finite difference method on a staggered grid system. A two-step projection method (Chorin, 1968, 1969; Lin and Liu, 1998), which has been proved to be very robust, is employed. The forward time difference method is used to discretize the time derivative. The convection terms are discretized by the combination of the central difference method and upwind method, while only the central difference method is employed to discretize the pressure gradient terms and stress gradient terms. The VOF method is adopted to track the free surface. Detailed numerical techniques are well reviewed in the other paper (Liu, 2007) and not repeated here.

Internal wave maker for the 2D NSE

For the 2D Navier-Stokes equation model with the VOF method, Lin and Liu (1999) proposed the internal wave maker using mass source function of the continuity equation.

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = s(x,z,t) \qquad (5)$$

where $s(x,z,t)$ denotes the nonzero mass source function which is defined in the source region Ω. The source region, Ω, is defined by a rectangular shape with the x-axis direction length, L_s and the z-axis direction length, H_s. When the width of the source region is small relative to the target wavelength, the generated wave can be regarded to start from the section where the center of the source region is located. If we further assume that all of the mass increase or decrease introduced by the mass source function contributes to the generation of the target wave, the following relationship between the source function $s(x,z,t)$ and the expected time history of free surface displacement $\eta(t)$ above the source region can be derived (Lin and Liu, 1999).

$$\int_0^t \int_\Omega s(x,z,t)\, d\Omega\, dt = 2\int_0^t C\eta(t)\, dt \qquad (6)$$

where C denotes phase velocity of the target wave.

Internal wave maker for the 3D NSE

The internal wave maker proposed by Lin and Liu (1999) is employed in the 3D Navier-Stokes equations model to generate a target wave in three dimensions. Eq. (5) can be extended to three dimensions as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = s(x,y,z,t) \qquad (7)$$

If we assume the generated wave start from the section where the center of the source region is located and the direction of the target wave propagation is parallel, the source function of eq. (7) can be regarded as the same function using in eq. (5). Therefore, we can easily generate several types of waves without modifying original functions or adding any terms on.

At the outgoing boundary a sponge layer is used to absorb the wave energy. The sponge layer is employed by adding a damping term to each momentum equation without pressure terms at the first step of a two-step projection method as follows:

$$\frac{\tilde{u}^n - u^n}{\Delta t} + u^n\frac{\partial u^n}{\partial x} + v^n\frac{\partial u^n}{\partial x} + w^n\frac{\partial u^n}{\partial x} = \upsilon\nabla^2 u^n - c_s u^n \qquad (8)$$

$$\frac{\tilde{v}^n - v^n}{\Delta t} + u^n\frac{\partial v^n}{\partial x} + v^n\frac{\partial v^n}{\partial x} + w^n\frac{\partial v^n}{\partial x} = \upsilon\nabla^2 v^n - c_s v^n \qquad (9)$$

$$\frac{\tilde{w}^n - w^n}{\Delta t} + u^n\frac{\partial w^n}{\partial x} + v^n\frac{\partial w^n}{\partial x} + w^n\frac{\partial w^n}{\partial x} = \upsilon\nabla^2 w^n - c_s w^n \qquad (10)$$

where $\tilde{u}^n$, $\tilde{v}^n$, $\tilde{w}^n$ represent intermediate velocities in each axis direction and c_s denotes the damping coefficient, which is defined as $c_s = \left[\exp(x/x_s) - 1\right] / \left[\exp(1) - 1\right]$ inside the sponge layer and set to zero outside the sponge layer. The sponge layer is the interval, $0 \le x \le x_s$, x_s is a length of the sponge layer (Li, 2008).

NUMERICAL SIMULATION

Table 1: Conditions of the numerical experiments

Items		Components
Water depth (d)		0.2m
Wave Height (H)		0.01m / 0.02m / 0.04m
Cell	X-axis direction	700 (non-uniform: 0.02m-0.06m)
	Y-axis direction	10 (uniform: 0.02m)
	Z-axis direction	65 (uniform: 0.004m)
	Total	455,000 (18m$\times$0.2m$\times$0.26m)

Solitary wave on a constant water depth

Numerical experiments are conducted to verify the internal wave maker in three dimensions. The solitary waves with different ratios of wave heights to local water depth are generated by the internal wave maker. Water depth is set to be constant and three different wave heights are employed. Table 1 shows conditions of the numerical experiments.

Figure 1 represents comparison of water surface profiles between numerical results and analytic solutions. As seen in the figure, the numerical model with the internal wave maker quantitatively matches well analytic solutions. Figure 2 shows the free surface profiles of internally generated solitary waves for all three cases. Solitary waves are generated at the center of the numerical domain and propagated symmetrically to left and right boundaries. By comparing to analytic solutions, numerical results are agreeable.

Solitary wave runup and rundown on a steep slope

Laboratory experiments of solitary wave runup and rundown on a steep slope have been performed by Lin *et al.* (1999) and detailed information is estimated by using PIV (particle image velocimetry) system. The experiments have been used to verify free surface tracking methods because stable and continuous runup and rundown are occurring simultaneously. Numerical simulations using the internal wave maker are performed in a domain with 4.49m $\le x \le$ 6.99m and -0.16m $\le z \le$ 0.11m. The computational domain is discretized by 350 uniform grids in x direction with Δx=0.01m, while in z direction, 95 uniform grids are used with Δz=0.003m. The time step size is dynamically adjusted during the computation to satisfy the stability constraints. Table 2 shows conditions of the numerical experiments and Figure 3 gives the

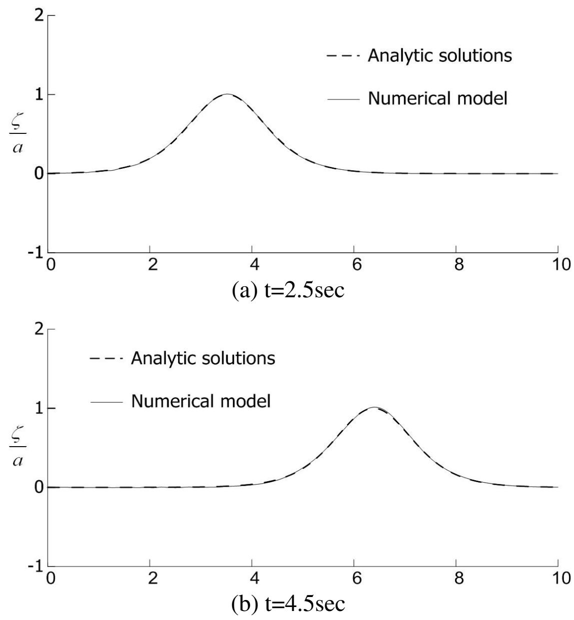

Figure 1. Comparison of numerical results and analytic solutions of solitary wave propagation on a constant depth (H/d=0.05)

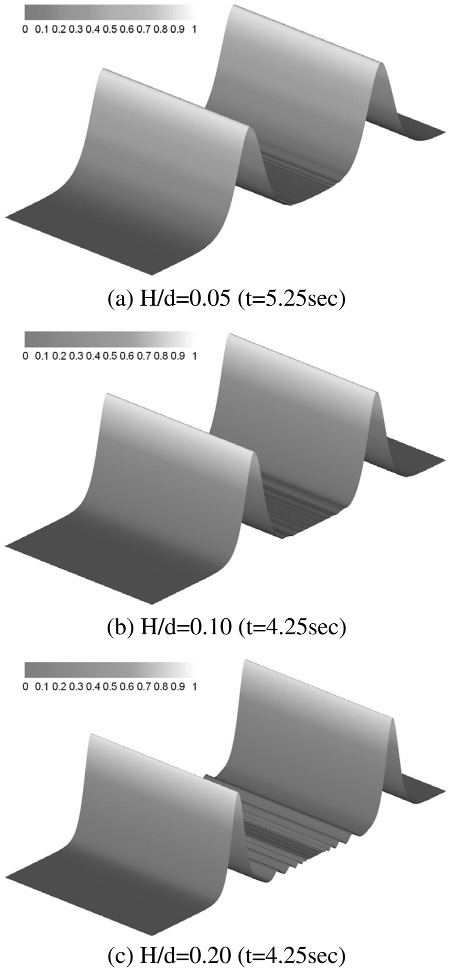

Figure 2. Propagation of internally generated solitary waves

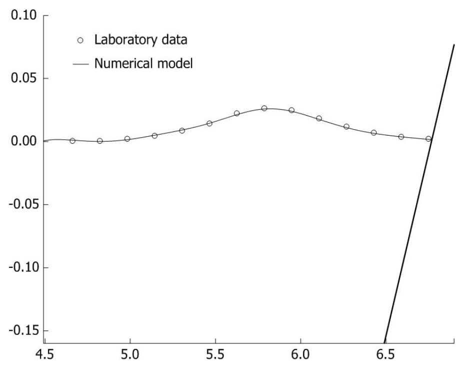

Figure 3. Computational domain of the numerical model and the comparison of free surface displacement (t=5.68sec)

Table 2: Conditions of the numerical experiments

Items		Components
Water depth (d)		0.16m
Wave Height (H)		0.027m
Cell	X-axis direction	350 (uniform: 0.01m)
	Y-axis direction	10 (uniform: 0.02m)
	Z-axis direction	95 (uniform: 0.003m)
	Total	332,500 (3.5m×0.2m×0.285m)

sketch of the computational domain and the measurement free surface at t=5.68sec. Figure 4 shows the free surface profiles of solitary wave runup and rundown on a steep slope. By comparing with laboratory observed data, numerical results represent well observed free surface profiles.

Solitary wave runup around a conical island

Laboratory experiments of solitary wave runup around a conical island were performed at the US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center (Liu *et al.*, 1995). The research has been well known a benchmark problem of runup simulations and provided a better understanding of the physical phenomena, such as wave refraction, diffraction

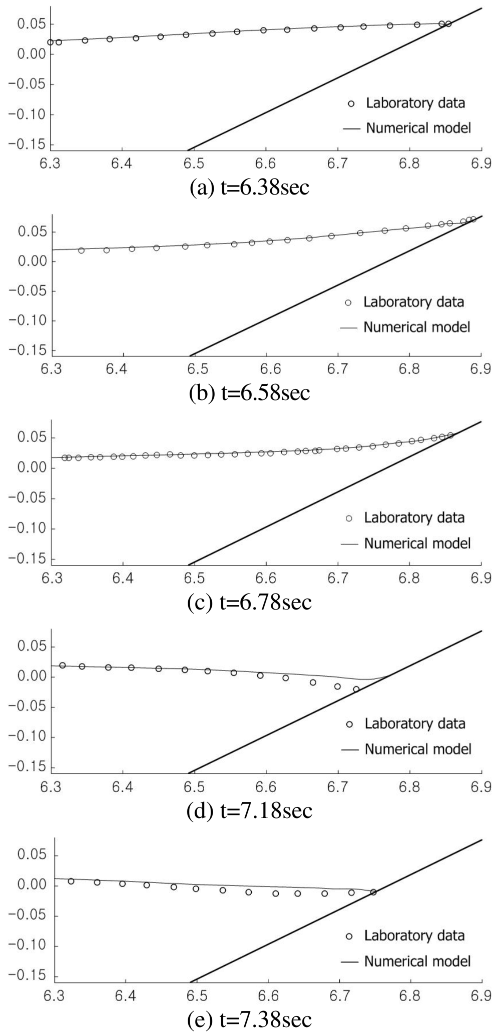

Figure 4. Solitary wave runup and rundown on a steep slope

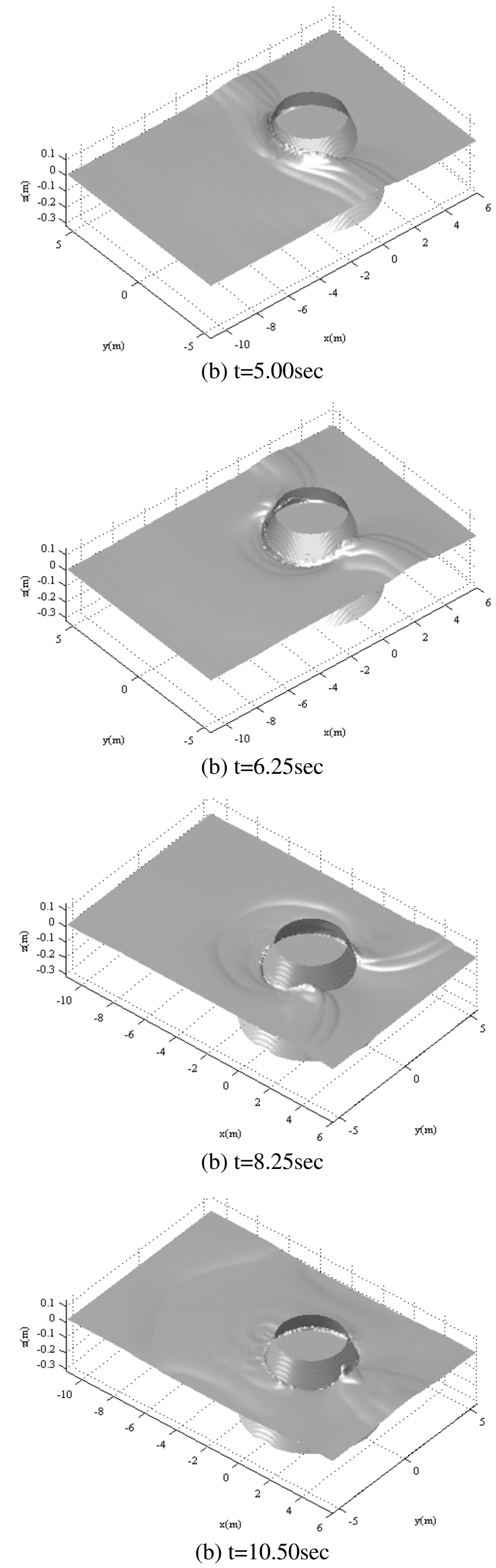

Figure 5. Solitary runup around a conical island

and breaking around the island. Therefore, the Navier-Stokes equations model with the internal wave maker is applied to simulate the laboratory experiment. Figure 5 shows numerical simulations around the island. Once a solitary wave approaches the island, large runup is occurred at the front side of the island. While a solitary wave passes the island, wave refraction and diffraction are occurred and then transformed waves are concentrated at the lee side. Numerical simulations represent well the physical phenomena. Therefore, the model can be regarded to simulate runup problems in three dimensions. A detailed quantitative analysis between numerical results and observed data has not been performed, however, additional researches are required to verify the model.

CONCLUDING REMARKS

In this study, the internal wave maker proposed by Lin and Liu (1999) is employed in the 3D Navier-Stokes equations model to generate a target wave. The solitary waves with different ratios of wave heights to local water depth are generated by the internal wave maker and compared with corresponding analytic solutions. Then, the model is applied to investigate solitary wave runup and rundown in two and three dimensions. Despite of small discrepancy, numerical results are reasonable and the internal wave maker can be applied to generate targeted waves internally in other numerical experiments.

The runup and rundown process of solitary waves around a variable topography have been attracted many researchers since they are similar to those of the tsunamis on coastal areas. Most of them have been conducted using depth-averaged numerical models or 2D Navier-Stokes equations model and numerical approaches using 3D Navier-Stokes equations models have been barely performed. Although application of the model is still in an early stage, the model can be regarded as a prospective efficient tool to analyze those problems.

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